PROTON-NUCLEUS SCATTERING AT 96 MEV

ARUNDHATI GHOSE

DEPARTMENT OF THEORETICAL PHYSICS,

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE, JADAVPUR, CALCUTTA-32

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ABSTRACT. In this paper the cross section for the proton-nucleus scattering at 96 Mev has been calculated in the Born approximation with a diffuse surface optical model potential

INTRODUCTION

The analysis of elastic scattering of protons by nuclei in the low and medium energy region has been done by Woods and Saxon and others. In earlier studies it has been found that for large angles a square well potential predicts larger values of scattering cross section than what are observed experimentally. With a view to obtaining a better fit of the theory with the experimental values, a rounding of the edge of the nuclear well was first introduced by Woods and Saxon (1954) who had obtained good agreement for scattering of 20 Mev protons by medium and heavy nuclei. Other workers have obtained similar agreements for different energy values of proton.

In the present paper using an optical model potential the proton-nucleus scattering cross section is calculated in the Born approximation for 96 Mev protons.

MATHEMATICAL FORMULATION

The scattering of proton by a nucleus is treated as scattering by point source generating an average potential. This potential consists of a nuclear optical model part plus a coulomb part arising from the uniform charge distribution of the nucleus. Thus

$$V = V_N + V_C$$

$$V_N = (V+iW)/[1+\exp\,(r-R)/a]$$

$$V_C = \frac{ze^2}{2R}\left(3-\frac{r^2}{R^2}\right) \quad \text{for} \quad r \leqslant R$$

$$= \frac{ze^2}{r} \qquad \text{for} \qquad r \geqslant R$$

where R and a are known as the half way radius and the diffuseness parameter respectively. R is expressed as

$$R = r_0 A^{1/3} \times 10^{-13} \text{ cm},$$

V and W are the parameters that determine the strength of the nuclear potential. In addition to the nuclear potential, we have the coulomb potential similar to that of uniformly charged sphere. The idea is to see how well such a velocity independent and central potential can explain the angular variation of the scattering cross section at high energies.

The scattering amplitude according to Born approximation is

$$f(\theta) = -\frac{8\pi^2 m}{h^2} \int_0^\infty \frac{\sin kr}{kr} V(r)\, r^2 dr$$

when
$$K = \frac{2mv}{\hbar} \sin(\theta/2)$$

The nuclear potential extends from zero to infinity and the coulomb potential is split into two parts,

$$f(\theta) = -\frac{8\pi^2 m}{h^2} \left[\int_0^\infty \frac{\sin kr}{kr} \frac{V+iW}{1+e^{(r-R)/a}} r^2 dr + \int_0^R \frac{\sin kr}{kr} \frac{ze^2}{2R} \left(3 - \frac{r^2}{R^2} \right) r^2 dr + \int_R^\infty \frac{\sin kr}{kr} \frac{ze^2}{r} r^2 dr \right]$$

$$= -\frac{8\pi^2 m}{h^2} \left[\frac{V+iW}{k} \left\{ \frac{a\pi R \cos kR}{\sinh ak\pi} - \frac{a^2\pi^2 \sin kR}{\sin h^2 aK\pi} \cdot \cosh ak\pi + 2Ka^3 \sum_{n=1}^{\infty} \frac{(-)^n \cdot n . e^{-\frac{nR}{a}}}{(n^2+K^2a^2)^2} \right\} + \frac{3ze^2}{R^3K^5} \sin KR - \frac{3ze^2}{R^2K^4} \cos KR \right]$$

The scattering cross section is given by

$$\sigma(\theta) = |f_V(\theta) + f_C(\theta)|^2 + |f_W(\theta)|^2$$

where $f_V(\theta)$, $f_W(\theta)$ and $f_C(\theta)$ are the scattering amplitudes due to nuclear and coulomb potentials.

The contribution of coulomb scattering predominates only at the forward scattering angles and sharply decreases with the increase of the angle. The interference due to the nuclear and coulomb potentials arises only from the real part of the nuclear potential and not from the imaginary part. The minima in the scattering cross section appear because of the sine and cosine terms present in the scattering amplitude arising from both the coulomb and neuclear potentials. It may be mentioned here that the positions of minima are almost entirely determined by r_0, the influence of a is very little. The values of the parameters in the interaction potential are so fixed as will give the best fit with experimental results As energy of the proton increases, it has been found necessary to decrease V and increase W. Finally at 300 Mev proton-nucleus scattering, Bjorklund and others (1957) have taken V to be zero and W equal to -16 Mev. We have tried to adjust the parameters such that the theoretical findings agree with the experimental results of Gerstein *et al* (1957) For silver we take $V = -19$ Mev, $W = 0$, $a = 0.5 \times 10^{-13}$ cm and $r_0 = 1.22 \times 10^{-13}$ cm We find that there is a minimum at 5° which is not borne out from observational data (figure 1). Moreover

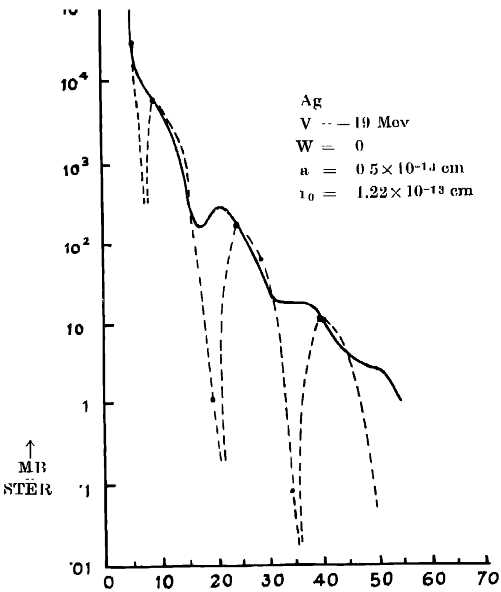
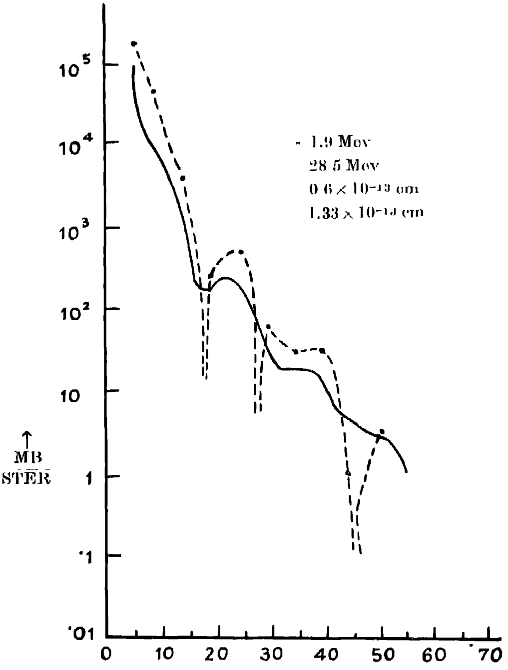


Fig. 1.

- - - - - Theoretical Curve

——— Experimental Curve

the positions of the minima are a little away from where they should be on the large angle side. In order to eliminate both the above anomalies for silver we propose to take $V = -1.9$ Mev., $W = -28.5$ Mev., $a = 0.6 \times 10^{-13}$ cm and $r_0 = 1.33 \times 10^{-13}$ cm. It is noticed (figure 2) that the theoretical values of the minimal points are still very much less than the experimental points. The values of the scattering cross section at large scatter- ing angles are appreciably affected by the variation of the parameter W of the nuclear potential.



c m. angle →

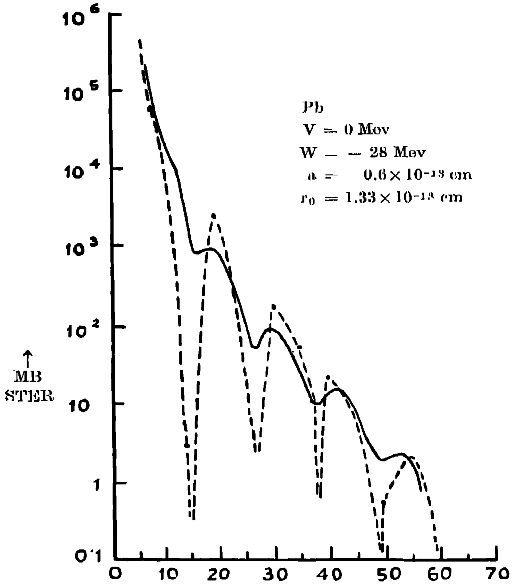
Fig. 2.

- - - - - Theoretical Curve

—— Experimental Curve

For lead we choose: $V = 0$, $W = -28$ Mev, $a = 0.6 \times 10^{-13}$ cm and $r_0 = 1.33 \times 10^{-13}$ cm. It is noticed (figure 3) that the positions of the minimum values of the cross section agree with the theory.

It may be mentioned here that in all the above cases, in the positions of minima the theoretical values drop to zero for the cross section whereas the experimental values are not as low as that. Since the Born approximation is not strictly valid in this energy region, the agreement between theoretical calculations and experi-



c.m. angle →

Fig. 3.

- - - - - Theoretical curve

———— Experimental Curve

mental values is not expected to be very close. We propose to improve the agreement by extending the calculations to higher energies and by modifying the potential.

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APPENDIX

With the substitution $\frac{r-R}{a} = x$ the integral

$$I = \int_0^\infty \frac{\sin kr}{kr} \frac{V}{1+e^{(r-R)/a}} r^2 dr$$

becomes

$$I = \frac{aV}{K}\left[\int_{-R/a}^{0} \frac{R+ax}{1+e^x} \sin K(R+ax)dx + \int_0^\infty \frac{R+ax}{1+e^x} \sin K(R+ax)dx\right]$$

Now
$$\frac{1}{1+e^x} = \sum_{n=0}^{\infty} e^{nx} (-)^n \quad \text{for} \quad r < R$$

and
$$\frac{1}{1+e^x} = \sum_{n=1}^{\infty} e^{-nx} (-)^{n+1} \quad \text{for} \quad r > R$$

The infinite series occurring in the expression for $f(\theta)$ can be summed up (c.f. Bromwich, 1947) as shown below.

Using
$$1+2a^2 \sum_{n=1}^{\infty} \frac{(-)^n}{n^2+a^2} = \frac{a\pi}{\sinh a\pi}$$

we get
$$\sum_{n=1}^{\infty} \frac{(-)^n}{n^2+k^2a^2} = \frac{1}{2a^2k^2}\left[\frac{ak\pi}{\sinh ak\pi} - 1\right] \quad \ldots \quad a)$$

Differenting (a) with respect to a, we have

$$\sum_{n=1}^{\infty}(-)^n \frac{1}{(n^2+k^2a^2)^2} = \frac{1}{2a^4k^4}\left[\frac{a\pi k}{\sinh ak\pi} - 1\right]$$

$$-\frac{1}{4a^3k^3}\left[\frac{\pi \sinh ak\pi - ka\pi^2 \cosh ak\pi}{\sinh^2 ak\pi}\right] \quad \ldots \quad (b)$$

Combining (a) and (b) we get

$$\sum_{n=1}^{\infty}(-)^n \frac{n^2}{(n^2+k^2a^2)^2} = \frac{1}{4ak} \frac{\pi \sinh ak\pi - ak\pi^2 \cosh ak\pi}{\sinh^2 ak\pi}$$

REFERENCES

Bjorklund, Blandford and Fernbach, 1957, *Phys. Rev.*, **108**, 795.
Bromwich, 1947, Introduction to the theory of Infinite Series, 2nd Edition
Gorstein, Niederer and Strauch, 1957, *Phys. Rev.*, **108**, 427.
Woods and Saxon, 1954, *Phys. Rev*, **95**, 577.